1 Constellation Model - 8 points

1.1
Suppose that we have image patches $I^j_i$ for each detected feature at location $x^j_i$ (i indicates the feature type and j the number of the detected feature). Suppose that we have conditional probability density functions $p(\text{good feature})$ and $p(\text{background clutter})$. Extend the 1-object 1-part model to take this information into account. Derive an expression for the likelihood ratio. In this formulation the appearance I and position x of the patches is assumed to be independent; give examples of situations where it is reasonable to assume this independence, and situations where x and I are clearly correlated.

1.2
Extend the multi-part object model to the case where all the parts look alike, i.e. we have a single detector which operates for all the F parts. In this case we obtain a single list of detected features rather than F separate lists.

2 Harris and Lucas-Tomasi-Kanade detectors. - 12 points

2.1 Lucas-Tomasi-Kanade detector
In class Pr.Perona described the Lucas-Tomasi-Kanade feature detector. This feature detector computes the matrix

$$H = \nabla I \cdot \nabla I^t = \begin{bmatrix} I_x^2 & I_x \cdot I_y \\ I_y \cdot I_x & I_y^2 \end{bmatrix}$$

and averages it over a small window:

$$\overline{H}(x, y) = \int_{\text{window}} H(x, y) \cdot dx \cdot dy$$

The locations of the local maxima of the smallest eigenvalue are selected as good features, provided they exceed an arbitrary threshold.

Compute the equation that gives the smallest eigenvalue $\lambda_2$ as a function of the trace and the determinant of $\overline{H}$, then implement it with Matlab. In order to calculate derivatives $I_x$ and $I_y$, you can convolve with the filter $[-1 \hspace{2mm} 0 \hspace{2mm} 1]$ and its transpose $[-1 \hspace{2mm} 0 \hspace{2mm} 1]^t$. Convolution is done with Matlab’s
function \texttt{conv2}. Typically, the integral that gives $H$ as a function of $H$ is computed by a convolution by a gaussian filter, and gaussian filters in the $x$ direction or in the $y$ direction can be approximated by $1/4 \ast [1 2 1]$ and $1/4 \ast [1 2 1]'$. Don’t forget to integrate over both $x$ and $y$!

You also have to choose a threshold under which local maxima of $\lambda_2$ will be rejected - this threshold is arbitrary.

How does this detector perform, what kind of features does it typically pick?

Note: in order to avoid problems due to aliasing, you may want to smooth the image a bit as a preprocessing step. This is done by a convolution of the same gaussian $1/4 \ast [1 2 1]$ in the $x$ and $y$ directions.

### 2.2 Harris detector

In file \texttt{harrisDetector88.pdf.gz} you will find the original paper describing the Harris detector. Notice that it uses the same matrix $H$ (this matrix is called 'second-order moment matrix'), but not the same 'recipe' to detect features.

Implement the Harris detector and compare it to the Lucas-Tomasi-Kanade one. The parameter $k$ at the bottom of page 150 is usually taken as $0.04$, but you can vary this value. Notice that the corresponding coefficient in the expression of $\lambda_2$ (Lucas-Tomasi-Kanade detector) is $0.25$. Which one performs better?

As always, illustrate those features detectors by a few examples. Don’t submits books of results, the TA won’t be happy.