Image Completion:
Filling in the Missing Data

Presented by:
Bernard Sarel
Lihi Zelnik-Manor

6/4/03
Agenda

• What do we mean by image completion?

• Problem definition – intuition into solutions

• Euler’s Elastica – a generalized model for inpainting

• Biological justification of inpainting processes

• Fast and Simple
Agenda

• What do we mean by image completion?

• Problem definition – intuition into solutions

• Euler’s Elastica – a generalized model for inpainting

• Biological justification of inpainting processes

• Fast and Simple
Image Completion
Resurrecting the art of great masters

"Cornelia, Mother of the Gracchi" by J. Suvee (Louvre).

What do we mean by image completion?
Image Completion
Resurrecting the art of great masters

What do we mean by image completion?
Image Completion
Restoration of Photographs

What do we mean by image completion?
What do we mean by image completion?
Completion of Textures

Generate “new” details

Copy information from other positions in image

What do we mean by image completion?
Agenda

• What do we mean by image completion?

• Problem definition – intuition into solutions

• Euler’s Elastica – a generalized model for inpainting

• Biological justification of inpainting processes

• Fast and Simple
Problem Definition

Given: - 2D finite domain $\Omega$,
- smaller domain $D \subset \Omega$

Required: - Estimate the information in $D$,
  based on the information in $\Omega$,

Constraints: - Overall information in the domain $\Omega$
  satisfies some “reasonable criteria”. 
“Reasonable Criteria”? 

- Pleasing to the eye 
- Smooth continuation of information across the gap 

Bertalmio et al
“Reasonable Criteria”?

- Model relies only on local features
Some Completion Methods

Learning

- Similarity criteria needed
- No information necessarily in image
- Appropriate for textures – not for structured images

Limiting, must learn similar images.

Function Interpolation

Matching Patterns (copying)

Complex for 2D shapes, hard to choose good criteria

Problem definition – intuition into solutions
What does intuition tell us?

Relevant information is near the boundaries

Continuing isophotes

Continuing edges & connecting them

Problem definition – intuition into solutions
Agenda

• What do we mean by image completion?

• Problem definition – intuition into solutions

• Euler’s Elastica – a generalized model for inpainting

• Biological justification of inpainting processes

• Fast and Simple
Image Inpainting – Two Approaches

Group 1:
Bertalmio, Sapiro, Caselles & Ballester

Approach:
• “Image Inpainting”

Group 2:
Chan, Shen, Kang

Approach:
• TV
• CDD

Chan, Shen, Kang:
Both approaches are joined via Euler’s Elastica model

Euler’s Elastica – a generalized model for inpainting
First group - Bertalmio et al.

Given:

We want to fill in:
Mathematical solution

Use an iterative process:

\[ I_{t+1} = I_t + \text{step} \cdot \frac{\partial I}{\partial t} \]

\( N = \text{Isophote direction} \)

\( \text{Lap} = \text{Smoothness estimator (Laplacian)} \)

\[ \frac{\partial I}{\partial t} = \nabla (\text{Lap}) \cdot \vec{N} \]

Bertalmio et al.
When do we stop?

\[ \frac{\partial I}{\partial t} = \nabla (\text{Lap}) \cdot \vec{N} = 0 \]
Some Results

Bertalmio, Sapiro, Caselles, Ballester
Some Results

Bertalmio, Sapiro, Caselles, Ballester
Denoising and Filling in

Bertalmio et al.
The full Monty (Python)

Bertalmio, Sapiro, Caselles, Ballester
Some Bizarre Results
Second Group - Chan & Shen

- Minimize gradients
- Allow step edges

A convenient approach

\[
\int_{D} f(|\nabla I|) \, dx dy
\]

\[
f(|\nabla I|) = |\nabla I|^\alpha
\]

How should we select \( \alpha \)?
Option A:

\[ \int |\nabla I|^\alpha \, dxdy = 10 \cdot 0.1^\alpha \]

Option B:

\[ \int |\nabla I|^\alpha \, dxdy = 0.5^\alpha \]

Solution depends on gap dimension.

Reasonable choice: \( \alpha = 1 \)
The TV model

To minimize:

\[ \int_{D} |\nabla I| \, dx \, dy \]

Steepest decent:

\[ I_{t+1} = I_{t} + \text{step} \cdot \text{change} \]

Propagation equation:

\[ \text{change} = \nabla \cdot \left( \frac{\nabla I}{|\nabla I|} \right) \]

Chan & Shen.
Hello! We are Penguin A and B. We can guys must think that so many words have made a large amount of image information loss. Is this true? We also disagree. We are more optimistic. The TV model can restore us. See ya!
But...

Does not maintain curvature:

Chan & Shen.
But...

Can break regions:

Reason: No penalty for high curvature.
Curvature Driven Diffusion
Chan, Shen, Kang

Solution: Minimize also curvature

Propagation equation:
\[
\frac{\partial I}{\partial t} = \nabla \cdot \left( \frac{f(|\kappa|)\nabla I}{|\nabla I|} \right)
\]

\(\kappa = \text{curvature}\)
TV vs. CDD

Original Image

Inpainting Domain

TV Inpainting

Curvature Inpainting

Chan & Shen.
How do the two approaches relate?
Mathematical Model

A general mathematical framework for non-texture Inpainting – Euler’s Elastica

\[ E[\gamma] = \int_{\gamma} (a + b\kappa^2) \, ds \]

Transforming to an area integral

\[ J[I] = \int \left( a + b\kappa^2 \right) |\nabla I| \, dx \, dy \]

Euler’s Elastica – a generalized model for inpainting
Functional, PDE, Diffusion, and all that…

Functional – we look for $I$ which minimizes $J$

Steepest descent search

$\frac{\partial I}{\partial t} = \nabla \cdot \vec{V}$

… which is a diffusion equation!

$J[I] = \int \left( a + b \kappa^2 \right) |\nabla I| \, dxdy$

Euler’s Elastica – a generalized model for inpainting
A look at how diffusion works

Euler’s Elastica – a generalized model for inpainting
Functional, PDE, Diffusion, and all that…

Functional – we look for \( I \) which minimizes \( J \)

\[
J[I] = \int \left( a + b \kappa^2 \right) |\nabla I| \, dx \, dy
\]

Steepest descent search

\[
\frac{\partial I}{\partial t} = \nabla \cdot \vec{V}
\]

… which is a diffusion equation!

Where:

\[
\vec{V} = \left( a + b \kappa^2 \right) \vec{g} - \frac{2b}{|\nabla I|} \frac{\partial \kappa |\nabla I|}{\partial \vec{n}} \, \vec{n}
\]

Euler’s Elastica – a generalized model for inpainting
Two Diffusion Directions

\[ \frac{\partial I}{\partial t} = \nabla \cdot \vec{V} = \nabla \cdot \left[ \left( a + b \kappa^2 \right) \vec{g} \right] - \frac{2b}{|\nabla I|} \frac{\partial \kappa}{\partial \vec{n}} \vec{n} \]

CDD Inpainting - Chan, Shen

\[ \frac{\partial I}{\partial t} = \nabla \cdot \left( f(|\kappa|) \vec{g} \right) \]

Along Normals

\[ f(|\kappa|) = a + b \kappa^2 \]

Bertalmio et al

\[ \frac{\partial I}{\partial t} = \nabla (\text{Lap}) \cdot \vec{N} \]

Transport information across isophotes

Transport information along isophotes

Generalized model for inpainting
CDD Inpainting - Chan, Shen

Bertalmio et al
Elastica’s Parameters \[ J[I] = \int (a + b\kappa^2) |\nabla I| \, dx \, dy \]

Original image with Inpainting domain

Euler’s Elastica – a generalized model for inpainting
To Be Honest…

If we propagate only along Isophotes…

Isophote lines can get mixed up.

… so Bertarlmio *et al* used alternating Isophote propagation and anisotropic diffusion
What Happens if…

… we have to consider ALSO noise …

\[ J[I] = \int_{D \cup B} \left( a + b \kappa^2 \right) \nabla I \, dxdy \]

\[ + \frac{\lambda}{2} \int_{B} (I - I_0) \, dxdy \]

Euler’s Elastica – a generalized model for inpainting
Denoising and Filling in

Elastica Inpainting - Chan, Shen

Euler’s Elastica – a generalized model for inpainting
TV + Noise Removal

\[ J[I] = \int_{D \cup B} |\nabla I| \, dx \, dy + \frac{\lambda}{2} \int_{B} (I - I_0) \, dx \, dy \]
Agenda

• What do we mean by image completion?

• Problem definition – intuition into solutions

• Euler’s Elastica – a generalized model for inpainting

• Biological justification of inpainting processes

• Fast and Simple
Age-related Macular Degeneration

- An ophthalmic (eye) condition characterized by progressive destruction and dysfunction of the central retina (macula).
What Patients See

Stargardt Macular Dystrophy is the most frequently encountered juvenile variant of macular dystrophy. This disorder was initially described in 1902 by Karl Stargardt, a German Ophthalmologist.

Patients usually present with Stargardt's first symptoms as a complaint of a decrease in legal vision generally within the first decade of life. Sometimes, instances of vision loss in patients may not become apparent until their 20s or 30s. Eyes of affected persons are generally not dilated initially. In the majority of cases, patients do not experience a loss in peripheral vision or night blindness. However, it has been shown that patients with Stargardt's disease have significantly peripheral problems. Once exposed to light, people with normal vision who enter a dark room are blinded by light or so blinded by a "spreneur". However, persons with Stargardt's disease may take 5, 10, or even 20 minutes to adjust to the dark environment.

The diagnosis of Stargardt Macular Dystrophy can often be made by observing characteristic changes that occur within the retina. These involve what has been described as an atrophic or "beaten-bronze" appearance of deterioration in the central portion of the retina that is referred to as the macula.

Biological justification for inpainting processes
What Patients See

Amsler Grid

Biological justification for inpainting processes
Zur & Ullman Experiment 1

Viewed

Perceived

Biological justification for inpainting processes
Zur & Ullman Experiment 2

Biological justification for inpainting processes
Zur & Ullman Experiment 3

Patients reported lower completion quality

Biological justification for inpainting processes
Mathematical Completion Model
(Zur & Ullman)

Known

Unknown
(to be completed)

Biological justification for inpainting processes
Mathematical Completion Model (Zur & Ullman)

- **Method 1**: Replace by Gaussian average
- **Method 2**: Replace by Gaussian average of known pixels
- **Method 3**: Replace by Gaussian average of C known pixels

Biological justification for inpainting processes
Image Completion by Zur & Ullman

Original image
Simulation of the corrupted retinal image
Gaussian smoothing (Method 1)
Adaptive size filling-in (Method 3)

Biological justification for inpainting processes
Texture Completion by Zur & Ullman

Input
Corrupted
Restored

Low performance for regular textures

Biological justification for inpainting processes
Agenda

• What do we mean by image completion?
• Problem definition – intuition into solutions
• Euler’s Elastica – a generalized model for inpainting
• Biological justification of inpainting processes
• Fast and Simple
Push-Pull
Gortler, Grzeszczuk, Szeliski and Cohen (Lumigraph)
Results *(Lumigraph)*

Fast and Simple
Results (Lumigraph)

Fast and Simple
Gaussian Filtering + Mask Enforcement

Known Mask

Iterate

Slower but gives similar results

Fast and Simple
Who is it?

Single scale
1000 convolutions
(all at highest resolution)

Multiple scales
260 convolutions
(80 at highest resolution)

Fast and Simple
Fast Digital Image Inpainting
Oliveira, Bowen, McKenna and Chang

\[
\begin{array}{ccc}
    a & b & a \\
    b & 0 & b \\
    a & b & a \\
\end{array}
\quad
\begin{array}{ccc}
    c & c & c \\
    c & 0 & c \\
    c & c & c \\
\end{array}
\]

\[a = 0.073235, \quad b = 0.176765, \quad c = 0.125\]
Add Diffusion Barriers

Fast and Simple
Examples

Fast and Simple
Results

Fast and Simple
Results

Bertalmio’s  Fast Inpainting

Fast and Simple
Image Inpainting - Not good for textures
Texture Completion – Not good for structures

Bertalmio et al
Handling Texture + Structure
Input = Structure + Texture

Bertalmio’s inpainting

Efros’ texture completion

Bertalmio et al
Is this a real picture?
(Wei & Levoy)
Agenda

• What do we mean by image completion?

• Problem definition – intuition into solutions

• Euler’s Elastica – a generalized model for inpainting

• Biological justification of inpainting processes

• Fast and Simple
Thank You